Computing the best coverage path in the presence of obstacles

Senjuti Basu Roy, Gautam Das, and Sajal Das

Outline

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2. Problem for opaque obstacles
3. Problem for transparent obstacles
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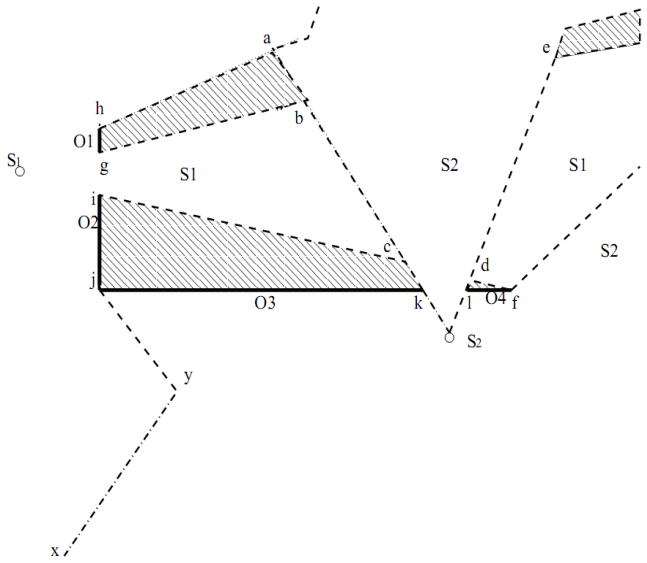
Problem formulation

- The cover value of a path from s to t is the maximum distance from a point of the path the its closest sensor.
- Best coverage path from s to t, BCP(s, t), is the path that has minimum cover value.
- Model: Set S of n sensors and set O of m line segment obstacles.
- Two types of obstacles:
  - Opaque obstacles: obstruct paths and the line of sight of sensors
  - Transparent obstacles: obstruct paths but allow sensors to see through them.

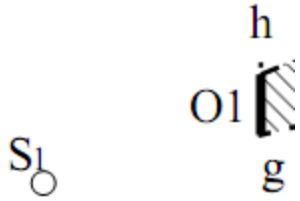
Problem for opaque obstacles

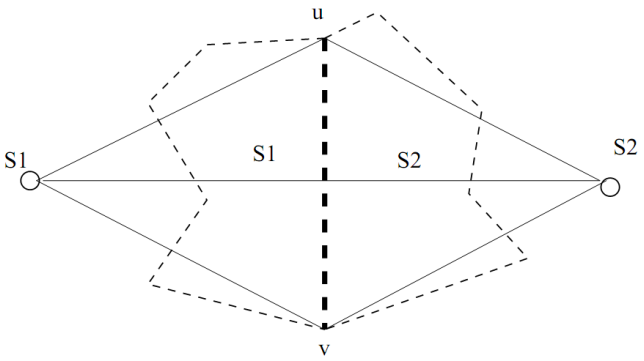
- Constrained weighted Voronoi diagram is a set of Voronoi cells $\{V(s_i) \mid s_i \in S\}$ such that $V(s_i) = \{x \in \mathbb{R}^2 \mid d_{wo}(x, s_i) \leqslant d_{wo}(x, s_j)$ and $d_{wo}(x, s_i) \neq \infty$ for all $s_j \in S, s_i \neq s_j\}$
- Observation: Each path go through a set of cells and intersects with them at the cell boundaries. The problem is solved if we can find the set of these intersections.

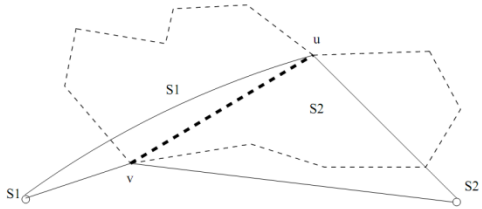


Problem for opaque obstacles

- There are three types of edges of CW-Voronoi Diagram:
  - (1) A part of obstacles
  - (2) A part of a perpendicular bisector between two sensors
  - (3) A part of an extension of a visible line
- Set of possible intersections:
  - Type 1, 2, 3: two ends of the edge.
  - Type 2: The intersection of the line formed by two sensors and the edge.







Dual graph of CW-Voronoi diagram

- Vertex set: set of sensors, the vertices of Voronoi diagram, s, t.
- Edge set:
  - For each edge (u, v) of Voronoi diagram that separate cells labeled by S_1 and S_2 , add four dual edges (u, S_1), (u, S_2), (v, S_1), and (v, S_2). If (u, v) intersect with S_1S_2 at T, add edges (S_1, T), (S_2 T).
  - For each edge (u, v) of type 1, which belongs to the cell labeled by sensor S, add two edges (u, S) and (v, S).
  - The weight is the Euclidian between two ends points.

Algorithm to compute BCP(s, t)

Algorithm 1 Calculate $BCP(S, O, s, t)$ for Opaque Obstacles

1: Use a known algorithm to construct the constrained Voronoi diagram of all n sensors and m obstacles (assign each sensor a weight of 1).

2: Construct the dual of this C-Voronoi diagram as described in Section 4.1.1.

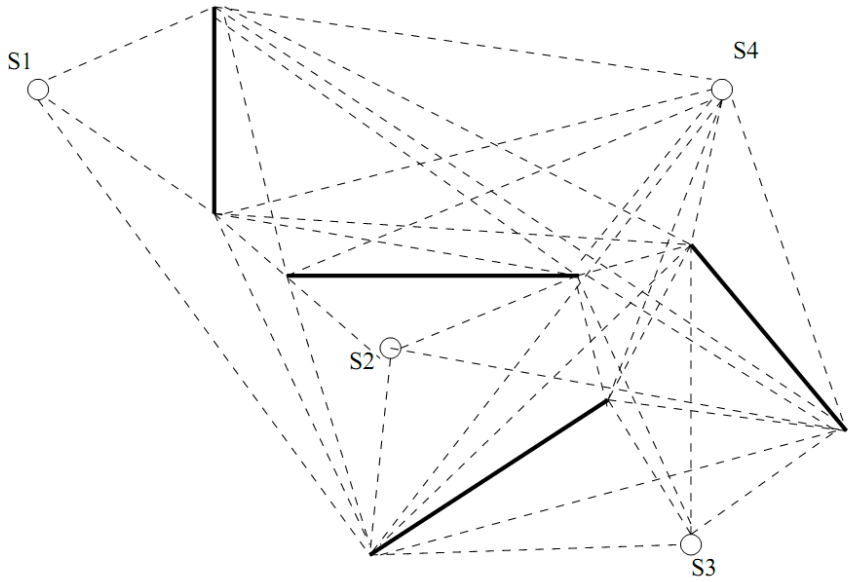
3: Run *Bellman-Ford* algorithm on this constructed dual graph starting at point s and ending at point t, which computes the *Best Coverage Path* between s and t.

4: The value of $Cover = \max(weight(e_1), weight(e_2), \ldots, weight(e_r))$ in the constructed path, where $e_1, e_2, \ldots, e_r$ are the edges in the best coverage path, $BCP(s, t)$.

- Time complexity: (1) takes $O(m^2n^2 + n^4)$ time and space to construct a CW-Voronoi diagram with $O(m^2n^2 + n^4)$ number of edges and vertices. Bellman-Ford algorithm at step 3 takes time $O((m^2n^2 + n^4)\log(mn + n^2)))$ .The total time is $O((m^2n^2 + n^4)\log(mn + n^2)))$.

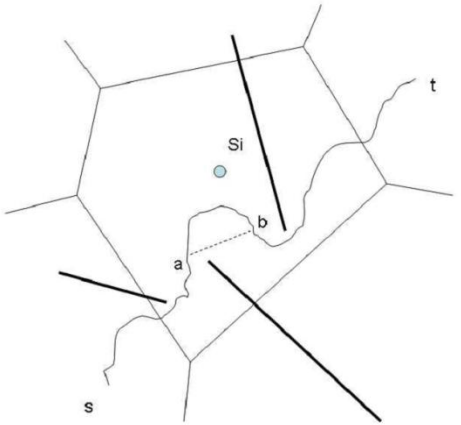
Problem for transparent obstacles

- Visibility graph o n locations is the graph of n vertices where there is an edge between a pair of vertices if they see each other.
- Observation:
  - At least a BCP is contained in visibility graph

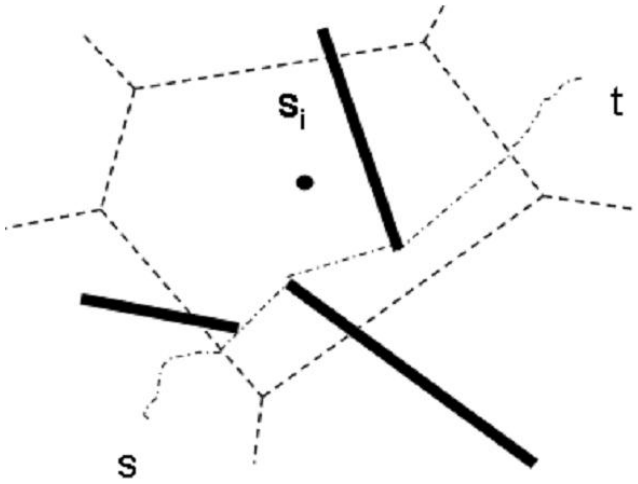


Problem for transparent obstacles

- A BCP which does not follow the visibility edges makes some bend either:
  - Type 1: Inside a Voronoi cell
  - Type 2: At a Voronoi bisector
  - Type 3: At a Voronoi vertex

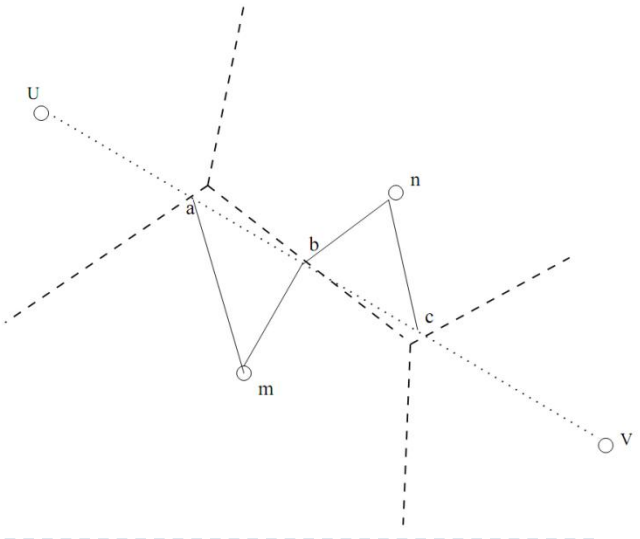


- Eliminate bends by replacing a arbitrary path from A to B by a line segment from A to B.



Problem for transparent obstacles

- Weight of visibility edge
  - Decompose an edge into separate line segments, each segment belong to a Voronoi cell.
  - Weight of each segment is Euclidian distance from the farther end to the corresponding sensor.
  - Weight of the edge is the max weight over all segments.



Algorithm computes BCP(s, t)

Algorithm 2. Calculate $BCP(S, O, s, t)$ for Transparent Obstacles

1: Construct the visibility graph of n sensor nodes, m line obstacles, point s and t.
2: Overlay a (normal) Voronoi diagram of the n sensor nodes on top of the created visibility graph.
3: Assign weight of each edge $e = (u, v) \in$ visibility graph
4: Run the Bellman-Ford algorithm on this weighted visibility graph starting at point s and ending at point t to compute a $BCP(s, t)$.
5: $cover = \max(weight(e_1), weight(e_2), \ldots, weight(e_r))$, where $e_1, e_2, \ldots, e_r$ are the edges of $BCP(s, t)$.

- Time complexity: Step 1 takes $O((m+n)\log(m+n)+x)$ to construct visibility graph where x is the number of visibility edges. Assigning weight to each edge takes O(n) time. Then step 3 takes $O(nm^2 + n^3)$. Step 4 finishes in $O((m+n)\log(m+n))$. Total time is $O(nm^2 + n^3)$.

Conclusion

- The algorithms requires to know exactly locations of all sensors, obstacles.
- Centralized algorithms.
- Does not solve the Maximum breach path problem.